MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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E. J. STONE, Esq., Vice President, in the Chair.

Dr. Hugo Gyldén, The Observatory, Stockholm;
Prof. Edward C. Pickering, Harvard College, U.S.;
Signor W. Tempel, The Observatory, Arcetri, Florence;
Prof. F. Tietjen, The Observatory, Berlin; and
Mons. F. Tisserand, Paris;

were balloted for and duly elected Associates of the Society.

The Geographical Position of the Observatory of the John C. Green School of Science at Princeton, N. Jersey, U.S.A. By Professor C. A. Young.

This Observatory was established in 1877 mainly for the purpose of instruction in Practical Astronomy. It is equipped with a $9\frac{1}{2}$-inch Equatoreal; a Meridian Circle of 4 inches aperture, with circles 2 feet in diameter; two Transit instruments of 3 and $2\frac{3}{4}$ inches aperture; and a 3-inch Prime Vertical Transit; also with the usual accessories of clocks, chronograph, micrometers, sextants, &c.

The longitude of the Meridian Circle was determined in 1878–79 by telegraphic connection with Washington. At that place the observations and reductions were made almost entirely by Asst. H. M. Paul; at Princeton the observations were made by myself, and the reductions were made for the most part by my assistant, Mr. M. McNeill, under my direction. The details of the work will appear in full in the "Washington Observations" for 1879.

Clock-comparisons were obtained on *seven* nights. After applying all corrections, that for my own personal equation being probably the most uncertain of them, these comparisons give the following results for the difference of Longitude between the *Dome of the Naval Observatory* at Washington and the Princeton *Meridian Circle.*

Date.		Δλ	Weight.
		m s	
1879, Jan.	18	−9 34·716	½
	22	34·491	1
	23	34·618	1
	30	34·556	1
March	5	34·401	½
	7	34·546	1
	12	34·438	1
Mean by weights ...		−9 34·538 ± ·021	

The discordances are larger than desirable, and are probably due mainly to my own unsteady personal equation, resulting from ill health at the time of the observations; partly also to a serious instability in the Y's of the Princeton Meridian Circle, which was not discovered and remedied until the work was nearly finished. It is believed, however, that the final result is substantially correct.

For the Latitude of the Observatory we have five closely-accordant determinations, by two different observers with three instruments. Four of the determinations were made by the Zenith-telescope method, and one by the Prime-vertical instrument.

Series I. consists of 82 observations of 28 pairs of stars on 9 nights, made by A. S. Flint in the early part of 1879, with the instrument known in the Observatory as the "Kahler Transit." It was made by Kahler of Washington, and has a "broken telescope" of $2\frac{3}{4}$ inches aperture and 31 inches focal length. It is provided with a reversing apparatus, with a delicate "latitude level" attached to the telescope tube, and with a declination micrometer; so that the instrument can be used as a zenith-telescope in the manner first proposed by Professor Lyman. It is in every way a most convenient and satisfactory instrument.

Series II. consists of 59 observations of 42 pairs, made by Mr. McNeill on two nights in May 1880.

Series III. consists of 114 observations of 49 pairs by the same observer and instrument on three nights in Sept. 1880.

Series IV. consists of 33 observations of 29 pairs by the same observer on two nights in Sept. 1880, made with the so-called Fauth Transit. This instrument, by Fauth and Co. of Washington, is a Transit of the ordinary form with a straight telescope of 3 inches aperture and 45 inches focus, carrying a diagonal

eyepiece and micrometer. It has a reversing apparatus, and a latitude level attached, not to the tube of the telescope, but to one of the 12-inch finding-circles on the axis. The field of view is rather restricted, and this renders the instrument much less convenient than the other for latitude work, though it is better for time determinations.

All of these observations were reduced under my direction by Mr. McNeill, and in substantially the same way. With approximate values of the micrometer constant and latitude, equations of condition were formed; the solutions of these equations by least squares furnished the necessary corrections to the assumed values. Thus the results depend entirely upon the star-declinations adopted and the observations themselves, without involving any subsidiary observations, made under different circumstances to determine the micrometer constant.

The declinations used were taken from the following Catalogues in the indicated order of preference.

1. Boss's Catalogue of 500 stars, published by the U.S. Northern Boundary Commission in 1878.

2. Auwers' "Fundamental Catalog" of 539 stars (Astron. Gesellschaft, 1879).

3. Safford's Catalogue of 2,018 stars, published by the U.S. War Department.

4. Catalogue of 2,179 stars contained in the Report of the U.S. Coast Survey for 1876.

The fifth series consists of 37 Prime-vertical latitude determinations from 11 different stars, made by Mr. McNeill in Jan. and Feb. 1881. The instrument is an old (but excellent) 3-inch Transit by Berge of London, fitted with an iron stand and reversing apparatus by Stackpole of New York, and with a striding level by Kahler.

26 of the 37 observations were made and reduced by Struve's method, the star being caught on both sides of the meridian; 15 of the 26 were observations of *α Lyræ*.

The pier of the Kahler instrument is in the same latitude as the Meridian Circle; the Fauth Transit is $0''\cdot025$ north, and the Prime-vertical instrument $0''\cdot186$ south. Applying the necessary corrections, the five results for the latitude of the *Meridian Circle* are as follow, viz.—

Series I.	$40^\circ\ 20'\ 57''\cdot791 \pm 0''\cdot148$
„ II.	$57\cdot763 \pm 0\cdot116$
„ III.	$57\cdot815 \pm 0\cdot067$
„ IV.	$57\cdot771 \pm 0\cdot173$
„ V.	$57\cdot775 \pm 0\cdot088$

The probable errors are deduced from the residuals of the equations of condition combined with the probable errors of the star declinations.

Giving each result a weight inversely proportional to the square of its probable error, and taking the weighted mean, we get as the astronomical Latitude of the Princeton Meridian Circle 40° 20′ 57″·792 ± 0″·044.

I am indebted to the Superintendent of the U.S. Coast and Geodetic Survey for data which determine our *Geodetic* Latitude and Longitude, when taken in connection with a survey made in 1877 under the direction of my colleague Professor McMillan, for the purpose of ascertaining our distance and direction from the cupola of the Theological Seminary, which is a Coast Survey point. "My Meridian Circle was found to be 2869^{ft}·8 east, and 1130^{ft}·9 north, of the seminary; giving a difference of longitude of 37″·04, and a difference of latitude of 11″·17."

According to the latest Coast Survey data, the geodetic Latitude and Longitude (from the meridian of Greenwich) of the Seminary cupola are respectively 40° 20′ 42″·842 and 74° 39′ 53″·942. For the astronomical Longitude of the Meridian Circle from Greenwich we get 74° 39′ 22″·47 by applying the observed difference of longitude (9^{m} 34^{s}·538) to the adjusted telegraphic longitude of the Washington Dome as adopted by the Coast Survey—viz. 77° 03′ 00″·54.

We thus have for the Princeton Meridian Circle the following data:—

	Latitude.	Longitude from Greenwich.
Astronomical ...	40° 20′ 57″·79	74° 39′ 22″·47
Geodetic	40 20 54·01	74 39 16·90
A−G	+ 3 78	+ 5·57

Reducing these deviations to a single one, it appears that gravity at Princeton is deflected towards the south-east by 5″·68 according to the Coast Survey determinations which are based upon Clarke's spheroid.

This, I understand, is perfectly in accordance with results obtained by the Coast Survey at other points in this region.

A line of levels carried down to the Delaware and Raritan Canal shows the axis of the Circle to be 214·5 feet above the mean sea level in Raritan Bay.

In order to prevent confusion it may be well to add that the *Halsted Observatory* in which the great Equatoreal of 23 inches diameter is soon to be mounted, is an entirely different building from the School of Science Observatory. It is about 2,000 feet distant to the west and north, but its position has not yet been carefully determined.

Princeton, N.J.:
1881, *May* 13.

Note on some Points connected with the Determination of the Coefficient of the Parallactic Inequality. By E. J. Stone, Esq.

There are three points raised by Messrs. Campbell and Neison in their paper "On the Parallactic Inequality" (*Monthly Notices*, vol. xli. No. 5), to which I should wish briefly to reply.

1. The correction 1″·56, which I have applied to the coefficient of the parallactic inequality deduced directly from a comparison between the Greenwich observations and Burckhardt's longitudes of the Moon, is the mean correction which is supposed to be required for errors in Burckhardt's tables when observations extending over nine years, and thus including a revolution of the apse, are grouped together; and Burckhardt's tables were used by me for the determination of the coefficient of the parallactic inequality in groups of nine-year periods. If Messrs. Campbell and Neison could show that this correction was in error 0″·5, for such a group of observations, they would have shown that the resulting value of the solar parallax was in error between 0″·03 and 0″·04. But these gentlemen have shown nothing of the kind; and I feel certain that no error, to this extent, has been committed through the neglect of any recognised lunar inequalities. I believe the correction applied was more than sufficiently accurate for the objects which I had in view. I did not expect to fix the value of the coefficient of the parallactic inequality, with the materials available in 1867, within 0″·25, and, although the result which Messrs. Campbell and Neison have recently found agrees very closely with that which I found in 1867, I am still very doubtful whether we do, as a matter of fact, know the value of the coefficient within a quarter of a second of arc.

2. I am afraid that Messrs. Campbell and Neison have hardly appreciated the force of the objection which I raised, and I think substantiated, against the logical completeness of their investigation in the *Monthly Notices*, vol. xl. Nos. 7 and 8.

I have, at present, seen no proof that the differences, between the optical semi-diameters of the Moon near the Full and Quarters, in the winter and summer months respectively, bear to each other a ratio of 0·25 to 0·75, or 1 : 3, rather than a ratio of 1 : 4, or 3 : 8; or 1 : 2. But waiving this point: what then? The investigation of Messrs. Campbell and Neison led them to the values of I, the supposed constant of irradiation, which I have given in the *Monthly Notices*, Nov. 1880, vol. xli. No. 1. These values of I are very nearly, indeed, as discordant as the values deduced from an investigation which Messrs. Campbell and Neison have most strongly condemned, simply on account of the discordant values of I to which it led. It appears to me, therefore, that the authors might with equal justice, and on the same grounds, have condemned their own investigation; and I entirely fail to see how

they have vindicated their investigation from the demurrer raised against it.

There is no doubt whatever that when we possess a perfect Lunar Theory, with all the six constants accurately determined, and are in possession of perfect methods for the reduction of our observations, including the elimination of all systematic errors, instrumental and personal, we shall free our equations of conditions from all discordances; all I maintain is that Messrs. Campbell and Neison have not yet succeeded in the elimination of such discordances, and that, as a matter of fact, their attempts to do so in this matter of irradiation have not, according to their own standard, been successful.

3. I am afraid that I must take exception to the statement made by Messrs. Campbell and Neison that "the entire series of the Greenwich meridian observations of the Moon unite in supporting the hypothesis of a forty-five-year inequality." To support this statement, the following results, as coefficients of the parallactic inequality corrected for the forty-five-year inequality, are given:—

"From observations

1851–1858	124″·23
1862–1869	124·09
1870–1876	124·13.

"Airy's value

1751–1846	124″·13."

These results undoubtedly strongly support the statement made; but the evidence brought forward will not, I fear, bear examination. The division of the group of observations 1862–1876, adopted by the authors, can afford no independent evidence for or against the forty-five-year inequality. But if we divide the observations into two groups of seven years each, with corrections for the inequality above the mean correction for the whole group, 1″·02, and equal to or below that mean, we shall find for the first group a value of the coefficient 125″·07, and for the second a value of 125″·20. These two results only differ by 0″·13, but if we introduce the correction for the supposed forty-five-year inequality, the results will be found to differ by 0″·39.

So far, therefore, as the results given by Messrs. Campbell and Neison for the values of the parallactic inequality can be trusted to afford evidence in seven-year groups, upon questions affecting a quarter of a second of arc, these results show that the value of the parallactic inequality found from the observations about 1870 does not require to be diminished by 1″·02. And, with respect to the results given for the observations 1851–1858, the authors have applied a correction for semi-diameter deduced as a correction to Hansen's semi-diameter, and

have apparently overlooked the fact that the observations 1851–1858 were either reduced directly with Adams' value of the semi-diameter, or corrected by the Astronomer Royal for the differences between the adopted values and Adams' value of the semi-diameter. As Adams' value is quite half a second larger than Hansen's value, the coefficient found by the authors requires to be increased from this cause by half a second. Nor is this all. In the paper, vol. xl. 1880, May and June, Nos. 7 and 8, the authors have increased the value of the parallactic inequality for error of Hansen's semi-diameter for the Transit Circle observations by $0''\cdot29$ in the mean group, whilst in the case of the observations 1851–1858 they have decreased the coefficient by $0''\cdot18$. These changes are certainly not satisfactory. So far as the observations 1854–1858 are concerned, which were made with the chronographic method, the same mean correction should be applied. And from a direct investigation I find that the correction required for error of adopted semi-diameter is $0''\cdot79$; which agrees very closely with the result above indicated, viz. :—

Excess of Adams over Hansen ...	$+0''\cdot52$
Correction to Hansen	0·29
	0·81

It appears to me that accepting all the authors' results except the correction to semi-diameter—which is certainly not satisfactory —the coefficient of the parallactic inequality from the observations 1851–1858, instead of agreeing with the result 1862–1876 after correction for the forty-five-year inequality is greater than that result by one second of arc, and is made more discordant by the application of the hypothetical inequality.

The value given for the earlier observations 1751–1846 (viz. $124''\cdot13$) is also to be received with caution.

The observations 1750–1810 give a value about $121''\cdot73$, whilst those from 1811–1851 give a value of $124''\cdot37$. The authors bridge over the difficulty presented by the large difference between these results by assuming that all the earlier observations require a constant correction of $1''\cdot6$; and this although there were several changes of instruments, observers, and methods of observing. Nor is this all. To reconcile the result thus obtained by this constant correction and the forty-five-year inequality, they have to assume that the result for the group of observations 1843–1851 requires to be diminished by $1''\cdot78$ for supposed errors of diameter. I cannot recognise the legitimacy of these steps. That the earlier observations, as reduced, show the existence of some systematic discordance with a period of about forty-five years was pointed out many years ago by the Astronomer Royal; but that such a discordance is due to an inequality in the theoretical determination of the Moon's longitude $=1''\cdot20 \sin [D-8(y-1825\cdot5)]$, appears to me most doubtful,

and is, I believe, quite unproven. It would not account for the known facts connected with these earlier observations. It fails to afford any explanation of the large constant differences between these earlier results and those more recently obtained. It fails to represent the results of the observations about 1843–1851. And it does not appear to be supported by the observations 1851–1876, and it is certainly not supported by the results derived from the Altazimuth observations.

Note on the Effect of Atmospheric Dispersion on the Determination of the Solar Parallax by Meridian Observations of Mars. By E. J. Stone, Esq.

There is one point connected with the determination of the Solar Parallax from meridian observations of *Mars* which has been frequently asserted, but has not, so far as I am aware, been yet proved. It is asserted that the larger value of the solar parallax which has been found by this method is due to atmospheric dispersion. And this point has again been raised by Mr. Gill in the abstract of his paper on the solar parallax (vol. xli. No. 6):—

"The large values of π which have been determined from the meridian observations of *Mars* are explained by the fact that, at the altitudes at which the planet was observed in Europe in 1877, the limbs of *Mars* are spread by chromatic dispersion of the atmosphere into a fringe of coloured light $2''{\cdot}2$ broad. For the upper limb the violet part of the fringe is projected outside the disk of the planet, and for the lower limb the red part of the fringe is outside. In the field of a meridian instrument illuminated by white light, more of the violet than of the red false limb would be obliterated, and hence the observer will cut more deeply with the spider-web into the false violet than into the more glaring red limb. Thus all the zenith distances measured would be too great, and the resulting parallax would also be too great."

The general effects of chromatic dispersion should, like those of refraction, be much diminished, or generally destroyed, when the position of the planet is referred both to stars of lesser and greater zenith distances, since the light of these stars also suffers dispersion.

And, if this cause was sufficient to produce an effect of $0''{\cdot}2$ on the resulting value of the solar parallax, it might certainly be expected to be distinctly shown in the difference between the measured vertical and horizontal diameters of *Mars*. The vertical diameters would, it appears to me, be very sensibly the greater. I am not aware of any well-established discordance of the kind. And, as a matter of fact, of the twenty-six diameters of *Mars* measured simultaneously at Greenwich in 1877 the mean difference of the diameters is only $0''{\cdot}25$, and the horizontal diameter happens to be the larger.

Note on the Inequality in the Moon's Latitude which is due to the Secular Change of the Plane of the Ecliptic. By Prof. J. C. Adams, M.A.

The first theoretical explanation of this inequality was given by Hansen in the year 1849, in No. 685 of the *Astronomische Nachrichten*, just a year after the Astronomer Royal had pointed out, in a letter published in the same journal—*Beilage zu No.* 648—that such an inequality was clearly indicated by the observations. In the same paper Hansen shows that there is a small term in the Moon's longitude depending on the same cause, the coefficient of which amounts to about $0''{\cdot}5$, the inequality being proportional to the cosine of the longitude of the Moon's node. The existence of this inequality also had been indicated by the Astronomer Royal from the observations, though he assigns to it a somewhat larger coefficient.

The calculation of both these inequalities is given by Hansen somewhat more fully in p. 491, art. 176 of his *Darlegung*.

In 1853 I communicated to Mr. Godfray a simple theoretical explanation of the inequality in latitude, which he inserted in his "Elementary Treatise on the Lunar Theory." This explanation is there given in rather too compendious a form, and I propose in the course of this paper to present to the Society the same investigation, with some slight modification, together with some additional remarks, which will, I hope, render it clearer than before.

At the meeting of the Society in March last, the Astronomer Royal gave an investigation of the inequality in latitude based upon the equations supplied by the "Factorial Tables" of his "Numerical Lunar Theory." About one portion of this investigation I wish to make a remark which seems to be important.

The Astronomer Royal forms his equations with reference to the *fixed* ecliptic, and, by integrating them, derives the value of the disturbed latitude above the *fixed* ecliptic, whence the latitude above the variable ecliptic is immediately deduced.

The latitude so found contains not only the inequality in latitude required, but also the small residual terms

$$Bt\{\cdot 003 \sin \lceil nt - C \rceil + \cdot 005 \sin \lceil nt - 2Nt + C \rceil\},$$

which the Astronomer Royal rejects, attributing them to accidental errors in the last places of the decimals employed.

I shall presently attempt to show that these terms must indeed be rejected, though not for the reason here supposed, but because they are destroyed by other terms which would be found by a more complete investigation.

It should be remarked that if terms of the above form really existed, they would, notwithstanding the smallness of their numerical coefficients, ultimately become much more important than the other terms in which t does not occur in the coefficients.

I propose to prove that in the complete solution of the differential equations no terms of the above-mentioned form can occur, supposing the displacements of the plane of the ecliptic to be proportional to the first power of t. The method which I employ for this purpose is the following.

Instead of solving the differential equations of motion with reference to the *fixed ecliptic* and then transforming the results so as to make them apply to the *variable ecliptic*, I first transform the differential equations of motion, so as to make them refer to the *variable ecliptic*, and when this is done, it is found that the terms which contain t in their coefficients disappear completely from the differential equations, so that the solution may be effected by the ordinary methods without any difficulty.

Employing the same data and notation as the Astronomer Royal, and taking into account only the terms which are independent of the Moon's eccentricity and inclination, I find

$$\delta s = -1''\cdot424\cos(nt-C)+0''\cdot048\cos(-nt+2Nt-C)-0''\cdot007\cos(3nt-2Nt-C).$$

The reason why, in the result found by the Astronomer Royal, the terms which are multiplied by t do not completely destroy each other, as they ought to do, appears to be the following.

It is at once seen, from the form of the periodic terms to which the Astronomer Royal confines his attention, that his investigation is only complete with respect to the terms which are independent of the eccentricity and inclination of the Moon's orbit. In order to take the eccentricity and inclination into account, other periodic terms must be included, the arguments of which involve the Moon's mean anomaly and its mean distance from the node. From the combination of these terms with each other will arise terms with the same *arguments* as those which are independent of the eccentricity and inclination, while each of their *coefficients* contains the square of one of these elements as a factor. Hence it is clear that terms of this order are omitted in the investigation.

On the other hand, a slight examination shows that the coefficients in the Astronomer Royal's expressions for

$$\frac{r}{a}\cos l \text{ and } v,$$

as well as in the quantities taken from his Factorial Table, include very sensible portions depending on the squares of the eccentricity and inclination.

In fact, it is plain that this must necessarily be the case since the quantities in question are functions of the Moon's *actual* coordinates, in which the numerical values of those elements are essentially involved.

Now, if terms depending on the squares of the eccentricity and

inclination were either wholly neglected, or completely taken into account, the terms which are multiplied by t would be found identically to destroy each other; but if, as in the present case, such terms are taken into account in one part of the investigation, and omitted in another part, it will follow that some of the terms multiplied by t will remain outstanding.

A curious circumstance relating to this inequality of latitude remains to be noticed.

In the *Mécanique Céleste*, tome 3, p. 185, Laplace proves that the plane of the Earth's orbit in its secular motion carries the plane of the Moon's orbit with it, so that the inclination of the Moon's orbit to the variable ecliptic is not liable to any secular variation.

In the same place he finds an analytical expression for the perturbation of latitude in reference to the variable ecliptic which is caused by the secular change in that plane.

Now the point to be noticed is that this analytical expression given by Laplace requires only the very slightest possible development to furnish for the inequality in question a result which is identical with the value given by the formula of Hansen, in which displacements of the ecliptic varying not only as the first but also as the second power of the time are taken into account. It is true that Laplace imagined that this inequality would turn out to be insensible, but this was only because he had not attempted to turn his formula into numbers.

Analysis.

I. Investigation of the inequality in the Moon's latitude which is due to the secular motion of the plane of the ecliptic, making the same suppositions and employing the same data as the Astronomer Royal.

At the time t let x, y, z be the rectangular coordinates of the Moon, and x', y' those of the Sun, referred to the Earth's centre as origin, the variable plane of the ecliptic at the same time being taken as the plane of $x\,y$.

Also at the time t let ξ, η, ζ be the rectangular coordinates of the Moon, and ξ', η', ζ' those of the Sun, taking the fixed plane of the ecliptic corresponding to $t=0$ as the plane of $\xi\,\eta$.

For greater simplicity we will suppose, with the Astronomer Royal, that the variable ecliptic intersects the fixed ecliptic in a fixed line, and that the angle between these two planes is proportional to the time.

Let this fixed line be taken as the axis of x and also as the axis of ξ, and let ωt be the angle between the variable and the fixed ecliptic, then the relations between the coordinates belonging to the two systems will be

$$\xi = x,$$

$$\eta = y\cos\omega t - z\sin\omega t,$$

$$\zeta = z\cos\omega t + y\sin\omega t,$$

and similarly

$$\xi' = x,$$
$$\eta' = y' \cos \omega t,$$
$$\zeta' = y' \sin \omega t.$$

Let r be the Moon's radius vector at time t, r' that of the Sun, m' the Sun's mass, μ the sum of the masses of the Earth and Moon, and R the disturbing function, then we have

$$\mathrm{R} = -\frac{1}{2}\frac{m'r^2}{r'^3} + \frac{3}{2}\frac{m'(\xi\xi' + \eta\eta' + \zeta\zeta')^2}{r'^5},$$

and the equations of motion, with reference to the fixed ecliptic, will be

$$\frac{d^2\xi}{dt^2} + \frac{\mu\xi}{r^3} = \frac{d\mathrm{R}}{d\xi},$$
$$\frac{d^2\eta}{dt^2} + \frac{\mu\eta}{r^3} = \frac{d\mathrm{R}}{d\eta},$$
$$\frac{d^2\zeta}{dt^2} + \frac{\mu\zeta}{r^3} = \frac{d\mathrm{R}}{d\zeta},$$

or, substituting the values of

$$\frac{d\mathrm{R}}{d\xi}, \quad \frac{d\mathrm{R}}{d\eta} \text{ and } \frac{d\mathrm{R}}{d\zeta},$$

(1) $$\frac{d^2\xi}{dt^2} + \frac{\mu\xi}{r^3} = -\frac{m'\xi}{r'^3} + \frac{3m'\xi'}{r'^5}(\xi\xi' + \eta\eta' + \zeta\zeta'),$$

(2) $$\frac{d^2\eta}{dt^2} + \frac{\mu\eta}{r^3} = -\frac{m'\eta}{r'^3} + \frac{3m'\eta'}{r'^5}(\xi\xi' + \eta\eta' + \zeta\zeta'),$$

(3) $$\frac{d^2\zeta}{dt^2} + \frac{\mu\zeta}{r^3} = -\frac{m'\zeta}{r'^3} + \frac{3m'\zeta'}{r'^5}(\xi\xi' + \eta\eta' + \zeta\zeta').$$

Now we have, from the values of η and ζ above given,

$$\frac{d^2\eta}{dt^2} = \left(\frac{d^2y}{dt^2} - 2\omega\frac{dz}{dt} - \omega^2 y\right)\cos\omega t - \left(\frac{d^2z}{dt^2} + 2\omega\frac{dy}{dt} - \omega^2 z\right)\sin\omega t,$$

and

$$\frac{d^2\zeta}{dt^2} = \left(\frac{d^2z}{dt^2} + 2\omega\frac{dy}{dt} - \omega^2 z\right)\cos\omega t + \left(\frac{d^2y}{dt^2} - 2\omega\frac{dz}{dt} - \omega^2 y\right)\sin\omega t,$$

and therefore

$$\frac{d^2\eta}{dt^2}\cos\omega t + \frac{d^2\zeta}{dt^2}\sin\omega t = \frac{d^2y}{dt^2} - 2\omega\frac{dz}{dt} - \omega^2 y,$$
$$\frac{d^2\zeta}{dt^2}\cos\omega t - \frac{d^2\eta}{dt^2}\sin\omega t = \frac{d^2z}{dt^2} + 2\omega\frac{dy}{dt} - \omega^2 z.$$

Now substitute for ξ, η, ζ and ξ', η', ζ' their values in terms of x, y, z and x', y' respectively, in

(1),

(2) $\cos \omega t + (3) \sin \omega t$,

(3) $\cos \omega t - (2) \sin \omega t$,

bearing in mind that

$$\xi\xi' + \eta\eta' + \zeta\zeta' = xx' + yy',$$

since each of these quantities represents

$$rr' \cos (r, r'),$$

and we have

$$\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} = -\frac{m'x}{r'^3} + \frac{3m'x'}{r'^5}(xx' + yy'),$$

$$\frac{d^2y}{dt^2} - 2\omega\frac{dz}{dt} - \omega^2 y + \frac{\mu y}{r^3} = -\frac{m'y}{r'^3} + \frac{3m'y'}{r'^5}(xx' + yy'),$$

$$\frac{d^2z}{dt^2} + 2\omega\frac{dy}{dt} - \omega^2 z + \frac{\mu z}{r^3} = -\frac{m'z}{r'^3},$$

which are the equations of the Moon's motion, with reference to the variable ecliptic.

The motion of the ecliptic is so slow (that is, ω is so small) that the terms involving ω^2 may be neglected.

We will now change the notation by writing for the Moon's coordinates $x + \delta x$, $y + \delta y$, and $z + \delta z$, instead of x, y, z respectively, in which expressions the new quantities x, y, z are taken so as to satisfy the equations of motion

$$\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} = -\frac{m'x}{r'^3} + \frac{3m'x'}{r'^5}(xx' + yy'),$$

$$\frac{d^2y}{dt^2} + \frac{\mu y}{r^3} = -\frac{m'y}{r'^3} + \frac{3m'y'}{r'^5}(xx' + yy'),$$

$$\frac{d^2z}{dt^2} + \frac{\mu z}{r^3} = -\frac{m'z}{r'^3},$$

which are those of the ordinary lunar theory, in which the motion of the ecliptic is not taken into account, so that x, y, z may be supposed to be known functions of t.

Hence the equations for determining the small increments δx, δy, δz of the coordinates, which are due to the motion of the ecliptic, are the following:

$$\frac{d^2\delta x}{dt^2} + \left(\frac{\mu}{r^3} + \frac{m'}{r'^3}\right)\delta x - \frac{3\mu x}{r^5}\left(x\delta x + y\delta y + z\delta z\right) = \frac{3m'x'}{r'^5}\left(x'\delta x + y'\delta y\right),$$

$$\frac{d^2\delta y}{dt^2} + \left(\frac{\mu}{r^3} + \frac{m'}{r'^3}\right)\delta y - \frac{3\mu y}{r^5}\left(x\delta x + y\delta y + z\delta z\right) - 2\omega\frac{dz}{dt} = \frac{3m'y'}{r'^5}\left(x'\delta x + y'\delta y\right),$$

$$\frac{d^2dz}{dt^2} + \left(\frac{\mu}{r^3} + \frac{m'}{r'^3}\right)\delta z - \frac{3\mu z}{r^5}\left(x\delta x + y\delta y + z\delta z\right) + 2\omega\frac{dy}{dt} = 0.$$

We may remark that no terms involving arbitrary constants need be added to the values of δx, δy, δz, since these may be supposed to be already included in the values of x, y, z.

Hence we may choose for δx, δy, δz any particular values which satisfy these differential equations, and we may consider these values to contain ω as a factor throughout.

If γ denote the sine of the mean inclination of the Moon's orbit, the value of z, and therefore that of $\frac{dz}{dt}$, will contain γ as a factor throughout. Hence the form of the first two of these differential equations shows that the values of δx, δy, found under the above conditions, will contain $\gamma\omega$ as a factor throughout, and therefore that the term

$$\frac{3\mu z}{r^5}\left(x\delta x + y\delta y + z\delta z\right),$$

which occurs in the third differential equation, will contain the factor $\gamma^2\omega$ throughout.

If, therefore, we neglect the square of γ, the equation for δz takes the simple form

$$\frac{d^2\delta z}{dt^2} + \left(\frac{\mu}{r^3} + \frac{m'}{r'^3}\right)\delta z + 2\omega\frac{dy}{dt} = 0.$$

Now let θ be the Moon's longitude at time t measured from the axis of x, that is from the line of intersection of the variable and of the fixed ecliptic.

Also let nt and $n't$ be the mean longitudes of the Moon and the Sun, omitting, for the sake of brevity in writing, the constants which always accompany nt and $n't$ respectively.

For the sake of simplicity, we will now neglect the eccentricities of the two orbits as well as their mutual inclination.

In this case we have, with abundant accuracy for our present purpose,

$$r = 0{\cdot}99911{,}92 - 0{\cdot}00717{,}34\cos 2(nt-n't) - 0{\cdot}00002{,}00\cos 4(nt-n't),$$
$$\theta = nt \qquad + 0{\cdot}01021{,}14\sin 2(nt-n't) + 0{\cdot}00004{,}24\sin 4(nt-n't),$$

where, as in my paper in the *Monthly Notices*, vol. xxxviii. p. 46, the angles are expressed in the circular measure, and the unit of distance is the mean distance in an undisturbed orbit which would be described by the Moon about the Earth in its actual periodic time.

Hence we have, as in the paper referred to—

$$\mu = n^2, \text{ and } \frac{m'}{r'^3} = n'^2.$$

Now choose the unit of time such that

$$n - n' = 1;$$

therefore, since in the case of the Moon

$$\frac{n'}{n} = 0{\cdot}07480{,}13,$$

we have

$$n' = 0{\cdot}08084{,}89,$$

and

$$n = 1{\cdot}08084{,}89.$$

From the values of r and θ above given, it is readily found that

$$\begin{aligned} y = r \sin\theta = {} & -0{\cdot}00868{,}79 \sin(-nt + 2n't) \\ & + 0{\cdot}99909{,}31 \sin nt \\ & + 0{\cdot}00151{,}43 \sin(3nt - 2n't) \\ & + 0{\cdot}00000{,}59 \sin(5nt - 4n't), \end{aligned}$$

and hence that

$$\begin{aligned} \frac{dy}{dt} = {} & + 0{\cdot}00798{,}55 \cos(-nt + 2n't) \\ & + 1{\cdot}07986{,}87 \cos nt \\ & + 0{\cdot}00466{,}54 \cos(3nt - 2n't) \\ & + 0{\cdot}00002{,}98 \cos(5nt - 4n't), \end{aligned}$$

and also, as in the paper referred to above,

$$\frac{\mu}{r^3} + \frac{m'}{r'^3} = 1{\cdot}17804{,}45 + 0{\cdot}02523{,}37 \cos 2(nt - n't) + 0{\cdot}00025{,}16 \cos 4(nt - n't).$$

Hence the equation to be solved becomes

$$\begin{aligned} & \frac{d^2\delta z}{dt^2} + \delta z\left[1{\cdot}17804{,}45 + 0{\cdot}02523{,}37 \cos 2(nt - n't) + 0{\cdot}00025{,}16 \cos 4(nt - n't)\right] \\ & + \omega\left[0{\cdot}01597{,}1 \cos(-nt + 2n't) + 2{\cdot}15973{,}7 \cos nt \right. \\ & \qquad \left. + 0{\cdot}00933{,}1 \cos(3nt - 2n't) + 0{\cdot}00006{,}0 \times \cos(5nt - 4n't)\right] = 0. \end{aligned}$$

Assume

$$\begin{aligned} \delta z = \omega[& c_{-3} \cos(-3nt + 4n't) + c_{-1} \cos(-nt + 2n't) + c_1 \cos nt \\ & + c_3 \cos(3nt - 2n't) + c_5 \cos(5nt - 4n't)], \end{aligned}$$

then, by substituting for δz and equating coefficients of similar terms, we have

$$\begin{aligned} -7{\cdot}34339{,}8c_{-3} + 0{\cdot}01261{,}7c_{-1} + 0{\cdot}00012{,}6c_1 & = 0, \\ 0{\cdot}01261{,}7c_{-3} + 0{\cdot}33320{,}6c_{-1} + 0{\cdot}01261{,}7c_1 + 0{\cdot}00012{,}6c_3 + 0{\cdot}01597{,}1\omega & = 0, \\ 0{\cdot}00012{,}6c_{-3} + 0{\cdot}01261{,}7c_{-1} + 0{\cdot}00981{,}0c_1 + 0{\cdot}01261{,}7c_3 + 0{\cdot}00012{,}6c_5 & \\ + 2{\cdot}15973{,}7\omega & = 0, \\ + 0{\cdot}00012{,}6c_{-1} + 0{\cdot}01261{,}7c_1 - 8{\cdot}31358{,}5c_3 + 0{\cdot}01261{,}7c_5 + 0{\cdot}00933{,}1\omega & = 0, \\ + 0{\cdot}00012{,}6c_1 + 0{\cdot}01261{,}7c_3 - 24{\cdot}63698{,}1c_5 + 0{\cdot}00006{,}0\omega & = 0. \end{aligned}$$

If we find the values of c_{-3}, c_3, and c_5 from these equations in terms of the two remaining coefficients c_{-1} and c_1, which can be advantageously done, since c_{-3} has a large coefficient in the first equation, c_3 in the fourth and c_5 in the fifth equation, we find

$$c_{-3} = 0{\cdot}00171{,}8c_{-1} + 0{\cdot}00001{,}72\ c_1,$$
$$c_3 = 0{\cdot}00001{,}5c_{-1} + 0{\cdot}00151{,}76\ c_1 + 0{\cdot}00112{,}2\omega,$$
$$c_5 = + 0{\cdot}00000{,}589c_1 + 0{\cdot}00000{,}3\omega,$$

and substituting these values in the 2nd and 3rd equations, they become

$$0{\cdot}33322{,}76c_{-1} + 0{\cdot}01261{,}74c_1 + 0{\cdot}01597{,}1\omega = 0,$$
$$0{\cdot}01261{,}74c_{-1} + 0{\cdot}00982{,}925c_1 + 2{\cdot}15975{,}1\omega = 0.$$

Whence again, we find

$$c_{-1} = 8{\cdot}69441\omega,$$
$$c_1 = -230{\cdot}8866\ \omega,$$

from which by substitution we obtain

$$c_{-3} = 0{\cdot}01097\omega,$$
$$c_3 = -\ 0{\cdot}34915\omega,$$
$$c_5 = -\ 0{\cdot}00136\omega.$$

Hence the solution of the differential equation for δz is

$$\delta z = \omega\{0{\cdot}01097\cos(-3nt+4n't) + 8{\cdot}69441\cos(-nt+2n't) - 230{\cdot}8866\cos nt - 0{\cdot}34915\cos(3nt-2n't) - 0{\cdot}00136\cos(5nt-4n't)\}.$$

Here ω is expressed in terms of the circular measure, and δz in terms of the unit of length defined before.

If s denote the sine of the Moon's latitude,

$$s = \frac{z}{r},$$

and if δs be the change in s due to the secular change in the plane of the ecliptic, we have

$$\delta s = \frac{\delta z}{r},$$

since

$$\delta r = 0,$$

according to the suppositions made above.

Also

$$\frac{1}{r} = 1{\cdot}00090{,}74 + 0{\cdot}00718{,}65\cos 2(nt-n't) + 0{\cdot}00004{,}58\cos 4(nt-n't).$$

Hence by substitution

$$\delta s = \omega\{0{\cdot}0369\cos(-3nt+4n't)+7{\cdot}8727\cos(-nt+2n't)-231{\cdot}0661\cos nt \\ -1{\cdot}1789\cos(3nt-2n't)-0\,0079\cos(5nt-4n't)\}.$$

Also s being supposed very small, δs is equal to the circular measure of the change of the Moon's latitude due to the secular change in the plane of the ecliptic, and if we divide δs by sin 1″ we shall find the change of the latitude in *seconds*

$$=\frac{\omega}{\sin 1''}\{0{\cdot}0369\cos(-3nt+4n't)+7{\cdot}8727\cos(-nt+2n't)-231{\cdot}0661\cos nt \\ -1{\cdot}1789\cos(3nt-2n't)-0{\cdot}0079\cos(5nt-4n't)\}.$$

Now, according to the data adopted by the Astronomer Royal, the circular measure of the angular motion of the plane of the ecliptic in 1 year is 0·479 sin 1″.

Also 1 year is represented in our notation by the time

$$\frac{2\pi}{n'}.$$

Hence

$$\frac{2\pi}{n'}\omega = 0{\cdot}479\sin 1'',$$

and

$$\frac{\omega}{\sin 1''} = 0{\cdot}479\,\frac{n'}{2\pi} = 0{\cdot}00516.354.$$

Therefore the inequality of latitude expressed in seconds is

$$0''{\cdot}0002\cos(-3nt+4n't)+0''{\cdot}0485\cos(-nt+2n't)-1''{\cdot}4242\cos nt \\ -0''{\cdot}0073\cos(3nt-2n't).$$

In this expression the mean longitudes nt and $n't$ are reckoned from the node of the variable ecliptic upon the fixed ecliptic. If the mean longitudes are reckoned from the equinox in the ordinary way, and if C be the longitude of the above-mentioned node, we must replace nt and $n't$ in the above by $nt-C$ and $n't-C$ respectively, and the expression for the inequality in latitude becomes

$$0''{\cdot}0002\cos(-3nt+4n't-C)+0''{\cdot}0485\cos(-nt+2n't-C) \\ -1''{\cdot}4242\cos(nt-C)-0''{\cdot}0073\cos(3nt-2n't-C).$$

In the above investigation the quantities ω and C are supposed to be constant. If these be subject to small secular variations, the differential equations become a little less simple, but are easily formed, and the above solution will require the following modifications, viz.—

(1) Instead of the constant value of ω we must employ the variable value which is of the form

$$\omega_0 + \omega' t;$$

(2) The coefficients of the above expression will be very slightly changed by quantities which are proportional to

$$\omega \frac{dC}{dt};$$

(3) The expression for the inequality of latitude will contain extremely small additional terms of the form

$$\frac{d\omega}{dt}\{g_{-3}\sin(-3nt+4n't-C)+g_{-1}\sin(-nt+2n't-C)+g_1\sin(nt-C)+g_3\sin(3nt-2n't-C)\};$$

that is to say, these terms will involve the *sines* instead of the *cosines* of the same arguments as before, and the coefficients of these new terms are proportional to

$$\frac{d\omega}{dt}.$$

II. Theoretical explanation of the same inequality, which was originally given, in substance, in Godfray's "Elementary Treatise on the Lunar Theory."

The general principle of this explanation may be very simply stated.

If, for a moment, we suppose the plane of the Moon's orbit to remain fixed, and imagine the plane of the ecliptic to turn through a very small given angle about a line in its own plane, this will give rise to corresponding small changes in the longitude of the Moon's node and in the inclination of the orbit to the ecliptic, and the magnitude of these changes will depend on the angular distance of the Moon's node from the line about which the ecliptic is supposed to be turning.

If now the planes of both orbits be supposed to vary continuously, the total changes in the longitude of the node and inclination of the orbit produced in an indefinitely small time will be found by adding together the changes respectively due to the motion of the plane of the ecliptic, and to the motion of the plane of the Moon's orbit with respect to the ecliptic when the latter is supposed to remain fixed during that small time. The motion last mentioned is given by the formulæ of the ordinary Lunar Theory, in terms of the disturbing force of the Sun. In consequence of the action of this force, the Moon's node gradually makes complete revolutions with respect to the line about which the ecliptic is turning, and the summation of all the momentary changes of node and inclination due to the motion of the ecliptic will produce periodic changes in those elements, the magnitudes of which, at any given time, like the momentary changes themselves, will depend on the angular distance, at that time, between the Moon's node and the line about which the ecliptic is turning.

The combined effect of these periodic changes in the position

of the node and in the inclination is to produce the inequality in latitude which is now under consideration.

The motion of the Moon's node is not uniform, but the principal inequalities by which that motion is affected have periods which are short compared with the time of revolution of the node.

Hence the periodic changes of node and inclination above described, will be accompanied by others which are due to the same cause, but which in consequence of the shortness of their periods will be comparatively unimportant, and the combined effect of these changes in the elements will be to add other terms which are equally unimportant to the expression of the inequality in latitude.

We proceed to find the analytical expressions for the changes in the longitude of the Moon's node and in the inclination of the orbit, due to the motion of the plane of the ecliptic, supposing the Moon's orbit itself to remain fixed.

Take C the longitude of the instantaneous axis about which the ecliptic is rotating at the time t,
ω the angular velocity of the ecliptic,
N the longitude of the Moon's node,
and i the inclination of the orbit, at the same instant.

Then, in the indefinitely small time δt, a point of the ecliptic situated in any arbitrary longitude L will move through an angular space

$$\omega \delta t \sin (L - C)$$

in a direction perpendicular to the ecliptic.

Hence the point of the ecliptic originally coincident with the node N will move through the space

$$\omega \delta t \sin (N - C)$$

perpendicular to the ecliptic.

And if δN be the consequent increase of the longitude of the node we have evidently from the figure,

$$\delta N = \omega \delta t \sin (N - C) \cot i$$

or

$$\frac{dN}{dt} = \omega \sin (N - C) \cot i.$$

Again, the point of the ecliptic 90° in advance of N will move through the space

$$\omega\delta t \sin (90° + N - C),$$

or

$$\omega\delta t \cos (N - C),$$

perpendicular to the ecliptic, and this quantity will measure the diminution in the inclination of the Moon's orbit.

Hence we have

$$\delta i = -\omega\delta t \cos (N - C),$$

or

$$\frac{di}{dt} = -\omega \cos (N - C).$$

Thus we have found the rates of change of the longitude of the Moon's node and of the inclination which are due to the motion of the ecliptic.

Now, suppose the formulæ which give the rates of change of the same two elements, with respect to a fixed ecliptic, which are due to the Sun's disturbing force, to be represented by

$$\frac{dN}{dt} = -c \cos i + F(\theta,\theta'),$$

and

$$\frac{di}{dt} = f(\theta,\theta'),$$

where $-c \cos i$ denotes the non-periodic term in $\frac{dN}{dt}$, c being approximately equal to $\frac{3}{4}\frac{n'^2}{n}$, and $F(\theta,\theta')$, $f(\theta,\theta')$ consist wholly of periodic terms which involve the longitudes θ, θ' of the Moon and Sun respectively, as well as the elements N and i.

Hence by what has been before said, if N′, i' denote the longitude of the node and the inclination at the time t, with respect to the variable ecliptic, $\frac{dN'}{dt}$ and $\frac{di'}{dt}$ will be given by the following formulæ:—

$$\frac{dN'}{dt} = -c \cos i' + F(\theta,\theta') + \omega \sin (N' - C) \cot i',$$

$$\frac{di'}{dt} = f(\theta,\theta') - \omega \cos (N' - C),$$

in which $F(\theta,\theta')$, $f(\theta,\theta')$ now involve the elements N′ and i', instead of N and i.

Now let N be the longitude of the node, and i the inclination at the time t, on the supposition that the ecliptic remains fixed,

all the other circumstances of the Moon's motion remaining unaltered; then we have as before

$$\frac{dN}{dt} = -c \cos i + F(\theta,\theta'),$$

$$\frac{di}{dt} = f(\theta,\theta').$$

Let

$$N' = N + \delta N.$$

and

$$i' = i + \delta i,$$

where δN and δi are entirely due to the motion of the ecliptic and therefore vanish with ω.*

Then neglecting the square of ω and supposing the value of θ, or the Moon's longitude, to remain unchanged, we have

$$\frac{d\,\delta N}{dt} = c \sin i\, \delta i + \left(\frac{dF}{dN}\right)\delta N + \left(\frac{dF}{di}\right)\delta i + \omega \sin (N-C) \cot i,$$

$$\frac{d\,\delta i}{dt} = \left(\frac{df}{dN}\right)\delta N + \left(\frac{df}{di}\right)\delta i - \omega \cos (N-C).$$

Now

$$\left(\frac{dF}{dN}\right), \left(\frac{dF}{di}\right),$$

and

$$\left(\frac{df}{dN}\right), \left(\frac{df}{di}\right),$$

are composed of periodic terms which have short periods compared with the time of revolution of the Moon's node—that is, with the period of the terms

$$\sin (N-C) \quad \text{and} \quad \cos (N-C).$$

Hence in integrating we may at first neglect the terms

$$\left(\frac{dF}{dN}\right)\delta N, \quad \left(\frac{dF}{di}\right)\delta i \quad \text{and} \quad \left(\frac{df}{dN}\right)\delta N, \quad \left(\frac{df}{di}\right)\delta i,$$

leaving them to be taken into account, if necessary, in a subsequent approximation.

For the same reason we may suppose cot i to be constant in integrating, and we make take

$$\frac{dN}{dt} = -c \cos i,$$

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* It is hardly necessary to mention that δN and δi are here employed in a wholly different sense from that in which the same symbols were used, for a temporary purpose, in the earlier part of this investigation.

omitting the periodic term $F(\theta,\theta')$; and we may also suppose that ω and C are constants.

With these simplifications, we have

$$\frac{d\,\delta N}{dt} = c \sin i\, \delta i + \omega \cot i \sin (N-C),$$

$$\frac{d\,\delta i}{dt} = \frac{\omega}{c \cos i} \cos (N-C) \frac{dN}{dt}.$$

From the latter of these equations

$$\delta i = \frac{\omega}{c \cos i} \sin (N-C),$$

and substituting this value of δi in the former, we find

$$\frac{d\,\delta N}{dt} = \omega \tan i \sin (N-C) + \omega \cot i \sin (N-C),$$

$$= \frac{\omega}{\sin i \cos i} \sin (N-C),$$

$$= \frac{\omega}{c \sin i \cos^2 i} \sin (N-C) \frac{dN}{dt},$$

and therefore

$$\delta N = \frac{\omega}{c \sin i \cos^2 i} \cos (N-C).$$

Now let NM be the Moon's orbit and M the place of the Moon as found from formulæ in which the plane of the ecliptic is supposed to be fixed, and let N′M′ be the Moon's orbit and M′ the place of the Moon at the same time taking into account the motion of the ecliptic.

Let $NM = \psi$, and $N'M' = \psi + \delta\psi$.

Also let s denote the sine of the Moon's latitude, and β the latitude itself, in the case when the ecliptic is supposed fixed;

And let $s + \delta s$ denote the sine of the latitude, and $\beta + \delta\beta$ the latitude itself, when the ecliptic is supposed to be variable.

Then

$$s = \sin i \sin \psi,$$

and

$$= \cos i \sin \psi\, \delta i + \sin i \cos \psi\, \delta\psi.$$

Now let us assume that MM′ is perpendicular to NM, in which case we shall have

$$\delta\psi = -\cos i\, \delta N,$$

and therefore

$$\delta s = \cos i \sin\psi\, \delta i - \sin i \cos i \cos\psi\, \delta N,$$

or substituting the values above found for δi and δN,

$$\delta s = \frac{\omega}{c} \sin\psi \sin(N-C) - \frac{\omega}{c\cos i} \cos\psi \cos(N-C).$$

But if θ denote the Moon's longitude, we have

$$\cos i \sin\psi = \cos\beta \sin(\theta - N),$$

and

$$\cos\psi = \cos\beta \cos(\theta - N).$$

Hence

$$\delta s = \frac{\omega}{c\cos i} \cos\beta \,[\sin(\theta-N)\sin(N-C) - \cos(\theta-N)\cos(N-C)],$$

or

$$\cos\beta\, \delta\beta = -\frac{\omega}{c\cos i} \cos\beta \cos(\theta - C),$$

and therefore

$$\delta\beta = -\frac{\omega}{c\cos i} \cos(\theta - C),$$

which is the inequality in latitude due to the motion of the ecliptic, expressed in the circular measure.

This value of $\delta\beta$ agrees exactly with that found in my article inserted in Godfray's Lunar Theory, since $c \cos i$ in this formula has the same signification as $\frac{I}{c}$ in Godfray, viz. the mean angular velocity of the Moon's node.

The steps, however, by which this result is arrived at, are slightly different in the two investigations. In the earlier one, the variation of $\cos i$ was neglected, and $\delta\psi$ was taken $= -\frac{\delta N}{\cos i}$, whereas in the present investigation the variation of $\cos i$ is taken into account, and $\delta\psi$ is taken $= -\cos i\, \delta N$, on the assumption that MM′ is perpendicular to NM.

It should be remarked that in both forms of this investigation, the neglect to take account of any variation of the Moon's radius vector and orbital longitude, due to the motion of the

ecliptic, may produce errors in the coefficient of the inequality in latitude which are of the order of the small quantity

$$\frac{\omega}{c} \sin^2 i,$$

so that the investigation is incompetent to decide such a question, for instance, as whether $\frac{\omega}{c \cos i}$ or $\frac{\omega}{c}$ is the more correct value of this coefficient.

The coefficient above found, expressed in seconds, is

$$\frac{\omega}{c \cos i \sin 1''}.$$

In order to evaluate this quantity numerically, we observe that $\frac{\omega}{c \cos i}$ is the ratio of two angular velocities: viz. the velocity of rotation of the plane of the ecliptic, and the mean angular velocity of the Moon's node; and in comparing these it is indifferent what unit of time is employed. According to the data adopted before, taking 1 year as the unit of time

$$\omega = 0{\cdot}479 \sin 1'', \quad \text{or} \quad \frac{\omega}{\sin 1''} = 0{\cdot}479.$$

Also since the Moon's node takes about 18·6 years to perform a complete revolution

$$c \cos i = \frac{2\pi}{18{\cdot}6} \text{ nearly.}$$

Hence

$$\frac{\omega}{c \cos i \sin 1''} = \frac{0{\cdot}479 \times 18{\cdot}6}{2\pi}, \quad \text{expressed in seconds,}$$

$$= 1''{\cdot}42,$$

which agrees with the value of the coefficient of the principal term found in the former investigation.

The form above found for $\delta\beta$ suggests a very simple geometrical interpretation of this inequality in latitude.

If we suppose a fictitious ecliptic to be inclined to the true ecliptic at the angle $1''{\cdot}42$, the circular measure of which is $\frac{\omega}{c \cos i}$, and if we also suppose that the longitude of its ascending node on the true ecliptic is

$$90° + C,$$

then the elevation of the fictitious above the true ecliptic corresponding to the longitude θ will be

$$= \frac{\omega}{c \cos i} \sin (\theta - 90^\circ + \mathrm{C}),$$

$$= -\frac{\omega}{c \cos i} \cos (\theta - \mathrm{C}),$$

$$= \delta\beta.$$

Hence the latitude above the fictitious ecliptic will be equal to β, that is, the expression for the Moon's latitude with respect to the fictitious ecliptic is the same as the expression found for the latitude in the case when the ecliptic is taken to be a fixed plane.

This geometrical interpretation of the inequality was first given by Hansen.

III. Note on the *Mécanique Céleste*, tome iii. p. 185 (edition of 1802).

At any arbitrary point whose longitude is λ, Laplace takes the elevation of the variable ecliptic above the fixed plane of reference to be represented by

$$\Sigma k \sin (\lambda + it + \epsilon),$$

and he shows that if s_1 denotes the perturbation of the Moon's latitude with respect to the variable ecliptic which is due to the motion of that plane,

Then

$$s_1 = \Sigma \frac{(2i + i^2)k \sin (\nu + i\nu + \epsilon)}{\frac{3}{2}m^2 - 2i - i^2},$$

where ν denotes the Moon's longitude;
or

$$s_1 = \Sigma \left[\frac{2ki}{\frac{3}{2}m^2} + \frac{4ki^2}{(\frac{3}{2}m^2)^2} \right] \sin (\nu + i\nu + \epsilon)$$

very nearly, neglecting i^2 compared with i except when it is divided by an additional power of $\frac{3}{2}m^2$.

Or, replacing $i\nu$ by it

$$s_1 = \sin \nu \, \Sigma \left[\frac{2ki}{\frac{3}{2}m^2} + \frac{4ki^2}{(\frac{3}{2}m^2)^2} \right] \cos (it + \epsilon)$$

$$+ \cos \nu \, \Sigma \left[\frac{2ki}{\frac{3}{2}m^2} + \frac{4ki^2}{(\frac{3}{2}m^2)^2} \right] \sin (it + \epsilon).$$

Now, Hansen's expression for the elevation of the variable

above the fixed ecliptic at any point whose longitude is λ is of the form

$$-p \cos \lambda + q \sin \lambda,$$

where p and q are functions of t, expressed in series of powers of t.

Comparing this with Laplace's expression for the same quantity, we have

$$-p = \Sigma k \sin (it + \epsilon),$$

hence

$$-\frac{dp}{dt} = \Sigma ki \cos (it + \epsilon),$$

and

$$\frac{d^2p}{dt^2} = \Sigma ki^2 \sin (it + \epsilon);$$

similarly

$$q = \Sigma k \cos (it + \epsilon),$$

$$-\frac{dq}{dt} = \Sigma ki \sin (it + \epsilon),$$

$$-\frac{d^2q}{dt^2} = \Sigma ki^2 \cos (it + \epsilon).$$

Hence, by substituting for $k \sin (it+\epsilon)$, $k \cos (it+\epsilon)$, &c. in Laplace's expression for s_1, their values in terms of p, q and their differential coefficients, we find

$$s_1 = \sin \nu \left[-\frac{1}{\frac{3}{4}m^2} \frac{dp}{dt} - \frac{1}{(\frac{3}{4}m^2)^2} \frac{d^2q}{dt^2} \right] + \cos \nu \left[-\frac{1}{\frac{3}{4}m^2} \frac{dq}{dt} + \frac{1}{(\frac{3}{4}m^2)^2} \frac{d^2p}{dt^2} \right],$$

which exactly agrees with Hansen's expression in his *Darlegung*, p. 490,* except that Hansen's argument $f+\omega-\theta_1$ represents the longitude on the orbit, whereas Laplace's argument ν is the longitude on the ecliptic; but these two longitudes may be employed indifferently in terms of the order of small quantities to which the approximation is restricted.

Laplace remarks that $\frac{3}{2}m^2$ is at least 4,000 times greater than $2i$, and he therefore infers that the above value of s_1 may be

---

* In this expression $\frac{dp}{dt}$ is equivalent to $b+b't$ in Hansen, and $\frac{dq}{dt}$ is equivalent to $c+c't$.

Also Hansen's expression $n(\alpha+\eta)$, which denotes the mean motion of the Moon's node, is equivalent to $\frac{3}{4}m^2$ in Laplace, as the latter takes n, the Moon's mean motion, to be equal to unity.

neglected as insensible. If, however, the numerical values of the quantities denoted by k had been known to Laplace, he would have seen that some of those values are very considerable, exceeding one degree, and therefore that $\frac{1}{4000}$ of this amount is by no means to be neglected.

Finally, we will reduce Laplace's transformed expression to a form immediately comparable with our former results.

The velocity perpendicular to the ecliptic of a point in any arbitrary longitude L is represented in one system by

$$-\frac{dp}{dt}\cos \mathrm{L} + \frac{dq}{dt}\sin \mathrm{L},$$

and in the other system by

$$\omega \sin (\mathrm{L}-\mathrm{C}).$$

Hence

$$\frac{dp}{dt} = \omega \sin \mathrm{C},$$

and

$$\frac{dq}{dt} = \omega \cos \mathrm{C};$$

$$\frac{d^2p}{dt^2} = \frac{d\omega}{dt}\sin \mathrm{C} + \omega \frac{d\mathrm{C}}{dt}\cos \mathrm{C},$$

and

$$\frac{d^2q}{dt^2} = \frac{d\omega}{dt}\cos \mathrm{C} - \omega \frac{d\mathrm{C}}{dt}\sin \mathrm{C}$$

Hence, putting c for $\frac{3}{4}m^2$, and denoting the Moon's longitude by θ as before, instead of Laplace's ν, we have

$$s_1 = \sin\theta\left[-\frac{1}{c}\omega \sin \mathrm{C} - \frac{1}{c^2}\left(\frac{d\omega}{dt}\cos \mathrm{C} - \omega\frac{d\mathrm{C}}{dt}\sin \mathrm{C}\right)\right]$$
$$+ \cos\theta\left[-\frac{1}{c}\omega \cos \mathrm{C} + \frac{1}{c^2}\left(\frac{d\omega}{dt}\sin \mathrm{C} + \omega\frac{d\mathrm{C}}{dt}\cos \mathrm{C}\right)\right],$$

or

$$s_1 = -\frac{\omega}{c}\cos(\theta-\mathrm{C}) - \frac{1}{c^2}\frac{d\omega}{dt}\sin(\theta-\mathrm{C}) + \frac{\omega}{c^2}\frac{d\mathrm{C}}{dt}\cos(\theta-\mathrm{C}),$$
$$= -\left(\frac{\omega}{c} - \frac{\omega}{c^2}\frac{d\mathrm{C}}{dt}\right)\cos(\theta-\mathrm{C}) - \frac{1}{c^2}\frac{d\omega}{dt}\sin(\theta-\mathrm{C}),$$

which is in accordance with the remark made at the close of investigation I.

Observations of the Solar Eclipse of 1880, *December* 31, *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The observations of this eclipse with the South-East Equatoreal were directed to the determination of the corrections to the Moon's R.A. and N.P.D. and to the semi-diameters of the Sun and Moon.

Putting

$d\alpha$ = correction to Moon's R.A.		$d\mathrm{A}$ = correction to Sun's R.A.	
$d\delta$ = " "	N.P.D.	$d\Delta$ = " "	N.P.D.
ds = " "	semidiam.	$d\mathrm{S}$ = " "	semidiam.,

the following arrangement of observations was drawn up, and the measures were made, by Mr. Christie, the declination microscopes being read by Mr. Criswick :—

(1) From $1^h\ 40^m$ to $1^h\ 55^m$ G.M.T., 10 differences of N.P.D. of N. and S. cusps, giving

$$(d\alpha - d\mathrm{A}) + \tfrac{1}{2}(d\delta - d\Delta) + ds + d\mathrm{S}.$$

(2) From $1^h\ 55^m$ to $2^h\ 15^m$ G.M.T., 10 differences of R.A. of p and f cusps, giving

$$d\alpha - d\mathrm{A} + ds + d\mathrm{S}.$$

(3) From $2^h\ 15^m$ to $2^h\ 40^m$ G.M.T., 13 differences of N.P.D. of S. limbs of Moon and Sun, giving

$$-(d\delta - d\Delta) - ds + d\mathrm{S}.$$

(4) From $2^h\ 40^m$ to $3^h\ 0^m$ G.M.T., 11 differences of N.P.D. of p and f cusps, giving

$$d\alpha - d\mathrm{A}.$$

(5) From $3^h\ 0^m$ to $3^h\ 25^m$ G.M.T., 12 differences of R.A. of p and f cusps, giving

$$-(d\alpha - d\mathrm{A}) + (d\delta - d\Delta) + ds + d\mathrm{S}.$$

Five resulting equations were found by taking the sums of the constituent equations in each group, obtained by a comparison of the measured with the tabular quantity as affected by the symbolical corrections $d\alpha - d\mathrm{A}$, $d\delta - d\Delta$, ds, and $d\mathrm{S}$:—

$-322''.79$ =	$+18''.094(d\alpha - d\mathrm{A})$ +	$11''.362(d\delta - d\Delta)$ +	$24''.570\ ds$ +	$24''.596\ d\mathrm{S}$
$-168''.73$ =	+ 9.797	− 0.821	+ 10.163	+ 10.512
+ 16.04 =		− 13.000	− 13.000	+ 13.000
$-159''.59$ =	+ 11.789	− 1.016	− 0.632	− 0.604
+ 100.18 =	− 10.376	+ 16.483	+ 22.997	+ 23.471

These being treated by the method of least squares gave the following four normal equations :—

$$
\begin{array}{rrrrr}
-10414''.46 = & +670\,02(d\alpha - d\mathrm{A}) + & 14\cdot54(d\delta - d\Delta) + & 298\cdot07\,ds + & 297\cdot37\,d\mathrm{S} \\
-\ 7056\cdot80 = & +297\cdot37 & +489\cdot32 & +1082\cdot30 & +1435\cdot72 \\
-\ 7449\cdot57 = & +298\cdot07 & +819\cdot52 & +1405\cdot23 & +1082\cdot30 \\
-\ 1924\cdot13 = & +\ 14\cdot54 & +571\cdot49 & +\ 819\cdot52 & +\ 489\cdot32
\end{array}
$$

the solution of which is

$$
\begin{aligned}
d\alpha - d\mathrm{A} &= -14''.23 \\
d\delta - d\Delta &= +\ 1''.17 \\
ds &= -\ 2\cdot72 \\
d\mathrm{S} &= -\ 0\cdot31
\end{aligned}
$$

From observations of the Sun with the Transit Circle on 1880 Dec. 9, 10, 22, 24, and 1881 Jan. 5, 6, 7, 10, 12, 15, the corrections to the Sun's tabular place are found to be

$$d\mathrm{A} = -1''.74; \quad d\Delta = -0''.44.$$

Combining this result with that obtained from the Equatoreal observations during the eclipse, we obtain for the corrections to the tabular place of the Moon in R.A. and N.P.D. :—

$$
\begin{aligned}
d\alpha &= -15''.97 = -1^{s}.06 \\
d\delta &= +\ 0''.73
\end{aligned}
$$

In reducing the observations, Bessel's Refractions of the *Tabulæ Regiomontanæ*, with Main's corrections below Z.D. 82°, have been used. Notwithstanding the low altitude of the Sun (3° 46′ at the last observation), the individual measures are very accordant, and there is no appreciable uncertainty in the refraction correction, which in no case exceeds 9″ in its differential effect on the two limbs or cusps observed.

Royal Observatory, Greenwich:
1881, *June* 8.

On the Moon's Photographic Diameter and on the Applicability of Celestial Photography to Accurate Measurement. By Prof. C. Pritchard.

(*Abstract.*)

The object of this communication is to investigate the amount of reliability assignable to Photography, as applied to the Moon; the enquiry being preliminary to a discussion of the nutation of the Moon's rotational axis, commonly called the Physical Libration, as obtained from measures made on Lunar Photographs.

1. In the first instance the instruments used are described and their scales or constants are determined.

2. The identity of the photographic and the ordinary telescopic image of the Moon for purposes of measurement is demonstrated.

3. The effects of inequalities on the Moon's limbs is shown to be inappreciable. This is demonstrated by measures taken from Triesnecker B or Ptolemy A to 74 points in succession, and without selection.

4. The effect of temperature on the general result is approximately determined.

5. The effect of the time of exposure of the photographic plate is ascertained.

6. Investigations are made respecting the effects of the Moon's altitude, and of the particular limb to points on which the measures are made.

7. The effect of any possible shrinkage of the photographic film is investigated.

8. The result of the entire set of enquiries shows that the photographic method is at least equal in point of accuracy to direct telescopic measures made on the actual disk of the Moon.

On the Comparison of the Computed and Observed Motions of the Components of the Binary System ξ Ursæ Majoris. By Prof. C. Pritchard.

About three years ago, in the first Fasciculus of the astronomical work done at the Oxford University Observatory, I published a brief account of a method which I had devised for the investigation of the relative orbits of binary stars. The method certainly offers peculiar facilities for such computations, and it seemed to me to hold out considerable promise at the time, especially as, owing to the wide discrepancies of ancient, and even of some modern, observations, abstruse and complicated mathematical processes appeared practically inapplicable. The method was, among other double stars, applied in particular to the case of *ξ Ursæ Majoris*, of which the elements are recorded in the Fasciculus referred to.

It is somewhat remarkable that Dr. Düner at Lund computed also an orbit for this system about the same time that the foregoing was computed at Oxford; that able astronomer employing an elaborate method wholly different from that devised by myself, yet an almost identity of elements was the result: and no doubt can now exist, that of *ξ Ursæ*, at least, the orbit is very approximately settled. The two sets of elements are as follows:—

	Oxford.	Düner.
Date of Perihelion Passage	1875·26	1875·29
Period of Revolution	60·80 yrs.	60·79 yrs.
Position of Node	100° 13′	101° 30′
Inclination of Orbit	56° 40′	56° 54′
Angle between Axis Major and Line of Nodes	235° 0′	234° 18′
Excentricity of Orbit	0·4159	0·3952
Semi-Axis Major	2″·580	2″·547

The following Table I. contains a comparison of the theoretical determination with observed facts during the last six years since the orbit was computed. The observed places herein given have been taken from the interpolation curve, drawn amongst the individual measures that have been hitherto published.

I.

Date.	Position Angle from Interpolation Curve.	Computed Position Angle (Oxford Elements).	Computed Position Angle (Düner's Elements).	C—O Oxford.	C—O Düner.	Distance from Interpolation Curve.	Computed Distance (Oxford Elements).	Computed Distance (Düner's Elements).	C—O Oxford	C—O Düner.
	°	°	°	°	°	″	″	″	″	″
1875·0	322·0	322·8	323·3	+0·8	+1·3	1·03	1·05	1·08	+0·02	+0·05
1876·0	309·1	307·7	309·1	−1·4	0·0	1·37	1·24	1·26	−0·13	−0·11
1877·0	298·5	296·6	298·6	−1·9	+0·1	1·54	1·43	1·44	−0·11	−0·10
1878·0	289·0	288·0	290·2	−1·0	+1·2	1·68	1·59	1·60	−0·09	−0·08
1879·0	281·0	280·8	283·2	−0·2	+2·2	1·78	1·71	1·72	−0·07	−0·06
1880·0	274·9	274·4	277·1	−0·5	+2·2	1·87	1·80	1·80	−0·07	−0·07
1881·0	269·0	268·5	271·3	−0·5	+2·3	1·88	1·85	1·85	−0·03	−0·03

The curve for the last few years is based on the observations given in Table II., in which may be seen the amount of divergence of the computed places from those measured by individual observers. For the earlier years, extending from 1819 to 1878, the comparisons of the observed with the computed places, founded on the Oxford Elements, are given in the Fasciculus referred to. The comparison of Dr. Düner's places will be found on page 197 of *Mesures Micrométriques d'Étoiles doubles* (Lund, 1876).

II.

Comparison with Oxford Elements.

Date.	Observed Angle.	Computed Angle.	Diff. C−O.	Observed Distance.	Computed Distance.	Diff. C−O.	Observer.
	°	°	°	″	″	″	
1877·26	294·9	294·2	−0·7	1·43	1·48	+0·05	Dembowski
77·34	293·9	293·5	−0·4	1·46	1·49	+0·03	Plummer
77·40	294·8	293·3	−1·5	1·52	1·50	−0·02	Wilson
77·47	294·2	292·3	−1·9	1·49	1·51	+0·02	Jedrzejewicz
78·25	286·9	286·1	−0·8	1·62	1·63	+0·01	Plummer
79·18	279·3	279·6	+0·3	1·70	1·73	+0·03	Plummer
79·18	278·1	279·6	+1·5				Gledhill
80·19	273·7	273·3	−0·4	1·84	1·81	−0·03	Plummer
80·20	272·0	273·2	+1·2	1·81	1·81	0·00	Gledhill
80·48	272·0	271·5	−0·5	1·81	1·83	+0·02	Jedrzejewicz
81·25	268·1	267·1	−1·0	1·84	1·86	+0·02	Plummer

Comparison with Dr. Düner's Elements.

1877·26	294·9	296·2	+1·3	1·43	1·49	+0·06	Dembowski
77·34	293·9	295·5	+1·6	1·46	1·50	+0·04	Oxford
77·40	294·8	295·0	+0·2	1·52	1·51	−0·01	Wilson
77·47	294·2	294·4	+0·2	1·49	1·52	+0·03	Jedrzejewicz
78·25	286·9	288·4	+1·5	1·62	1·63	+0·01	Oxford
79·18	279·3	282·1	+2·8	1·70	1·74	+0·04	Oxford
79·18	278·1	282·1	+4·0				Gledhill
80·19	273·7	276·0	+2·3	1·84	1·82	−0·02	Oxford
80·20	272·0	276·0	+4·0	1·81	1·82	+0·01	Gledhill
80·48	272·0	274·3	+2·3	1·81	1·83	+0·02	Jedrzejewicz
81·25	268·1	270·0	+1·9	1·84	1·86	+0·02	Oxford

In 1825, Sir John Herschel remarks that this star merits "constant and careful observation." Now that so close an approximation has been made to the representation of the actual facts, the interest attaching to such questions has unquestionably increased.

Before concluding this communication I think it my duty to the Society to record the great practical assistance which I have derived from Sir John Herschel's Reference Catalogue of 10,000 double stars, published in vol. xl. of their *Memoirs*. The chief utility of such Catalogues relates to their furnishing materials for the computations of orbits, and in that volume there is an asterisk or distinguishing mark placed against every binary star of which an orbit has ever yet been computed, or, for some years to come, is likely to be computed. The asterisk indicates that Sir John Herschel bequeathed to the Society a collection of papers, accessible to astronomers and containing a well-arranged and complete account of every observation he could find of the double star in question. It extends to every star in W. Struve's *Mensuræ Micrometricæ*, and forms a noble bequest and invaluable repertory. At some future time there may be a few other double stars, of which it may be worth the trouble and expense of the Society to collect a complete series of observations; but at present W. Struve's binaries are far from exhausted. In reference to this intrinsically valuable Catalogue, the last work of the great astronomer's declining years, I may mention that, on comparing the new edition of *Admiral Smyth's Cycle* with the Reference Catalogue in vol. xi., I found (with one solitary and very intelligible exception) every double star that is contained in the Cycle, recorded and referred to also in the Catalogue, up to the date of its completion.

Oxford:
1881, *June* 8.

The Planetary Nebula in Cygnus (*R.A.* $21^h\ 2^m$, *Decl.* 47° 22′).

By S. W. Burnham, Esq.

Astronomers interested in nebulæ seem to have overlooked the fact that the Planetary Nebula found by Dr. Copeland at the Dunecht Observatory last November was discovered a good many years ago. I picked it up July 6, 1873, with the 6-inch refractor, and described it in *Monthly Notices* of the Royal Astronomical Society for December 1873 (vol. xxxiv. p. 71) as "a very remarkable and curious double or elongated planetary (?) nebula. It is close to a 9·3 mag. star, *Arg.* (47°)3289. P=245°; D=30″. It bears magnifying well, and the elongation could not be seen with less than 400. With a large aperture it would probably be a very interesting object."

Subsequently, in 1878, it was examined with the 18½-inch

refractor of the Dearborn Observatory, and the nuclei measured with reference to each other, and to the 9·3 mag. star. These observations will be found in *Memoirs* of the R.A.S. vol. xliv. pp. 78, 157. It will be noticed that my measures of the nuclei disagree entirely with those of Dr. Copeland (*Urania*, Jan. 1881).

Burnham	P = 88°·5	D = 2″·6	1878·47
Copeland	71·8	8·0	1880·91

I cannot but think there is considerable error from some source in the last distance; for between 1873, when, as stated above, a high power was necessary to show the double character of the nebula, and 1878, when my measures were made, there was no apparent change whatever; and I am quite confident I looked at it at least once in 1880 and found its appearance substantially the same. A re-examination, however, will at once show what the real or approximate distance is.

Whatever the merits or advantages of the spectroscopic plan for the discovery of nebulæ may be generally, certainly no new method is required for this particular object; for any good refractor of four or five inches aperture is quite sufficient to detect it at once with an ordinary eyepiece.

Chicago:
1881, *March* 8.

The Nebula near Merope. By G. W. Hough, Esq., and S. W. Burnham, Esq.

The observations already made in regard to Tempel's Nebula in the *Pleiades*, although very numerous, are nevertheless so discordant that additional facts may be of interest. In Herschel's Catalogue the description is as follows: exceedingly interesting object; bright; very large; irregular figure; variable. From this description it would be inferred that it was a comparatively easy object.

The casual examination of *Merope* and its immediate neighbourhood on a number of nights during 1879 and 1880 with the 18½-inch Clark Refractor, with powers 120 and upwards, failed to show any nebulosity whatever.

The recent very positive statement of Tempel in the *Monthly Notices*, vol. xl. No. 9, in which the Nebula is so sharply delineated, led us to make a thorough examination of *Merope* with the Chicago Refractor, using various powers and apertures, to satisfy ourselves with regard to its existence.

In order that the circumstances affecting its visibility should be the most favourable, only first-class observing nights were chosen for making these examinations.

The following brief notes give the results of our work in this direction.

Nov. 29, 1880. Seeing good. Comet eye-piece, negative. Power 60; object-glass 12 inches, and full aperture.

When *Merope* was placed near the edge or outside the field, a faint glow was seen, south and preceding, in the general direction corresponding to Tempel's drawing, but nothing which would ordinarily be regarded as a nebula.

Dec. 2, 1880. Seeing excellent. The gauze ring of *Saturn* very sharply defined. Positive eye-piece. Power 120.

When *Merope* was placed near the edge of the field a faint glow was seen similar to that observed on Nov. 29th. A narrow strip of tinfoil was then placed across the field to cut off the direct light of *Merope*, when the glow disappeared, and no trace of nebulosity was visible.

Jan. 30, 1881. Seeing good. *Jupiter* and *Saturn* would bear a power of 638. Positive eye-piece. Power 120.

With a clear field, no glow or nebulosity could be seen. An occulting-bar was then used to cut off the light of *Merope*, with the same result.

March 22, 1881. Seeing good. Comet-eye-piece. Powers 60 and upwards. No trace of nebula was visible. Immediately after making this examination the telescope was set for the following faint Nebulæ, all of which proved to be comparatively easy objects: Dreyer and Herschel's, Nos. 5381, 5388, 5399, 5400. 5415, 5416, 1661.

The observation on November 29 led us to conclude that possibly the faint glow was due to nebulous matter, but the subsequent observation, in which an occulting-bar was used, proved conclusively that it proceeded from *Merope*.

With higher powers, no trace of nebula was ever seen in the neighbourhood of *Merope*.

So many observers, however, with various kinds of telescopes, have obviously seen something *near* this locality, that the question is one of more than ordinary interest, and the negative testimony of any one instrument should have but little weight, provided the positive evidence is sufficiently concordant. Whether, however, observers have seen Tempel's Nebula or something else will best be ascertained by comparing all observations with Tempel's description and drawing. A Nebula seen in any other part of the *Pleiades* cannot, therefore, be regarded as the object under discussion.

Tempel says: "It is now ascertained beyond question that the Nebula exists. It has been observed with the great Washington Refractor, and again with the Reflector of Lord Rosse."

In how far this statement is warranted may be ascertained from the following facts :—

"Washington Observations," 1874, p. 75.

"Tempel's Nebula in the *Pleiades* has been viewed occasionally."

"Wash. Obs." 1878.

"1875, Oct. 25. Power 158. Observer, Holden.

"*Merope* is a nebulous star, with a dark space on its *preceding north* and *following* sides. To the *south* it is joined by nebulosity to the main body of nebulosity above. This is not uniform in brightness, but cut up into portions by dark channels, several of them *north*, and following *Merope*. The nebulosity extends north [south] of this star some distance, some 6′ or 7′ at least, and perhaps more, as this point was not looked to. With 400, *Merope* is seen nebulous, but the rest is lost."

"Comet-seeker: power [?].

"I have drawn a dotted line on Tempel's map (pub. "R. O. di Milano," No. 5, plate ii.) where I see the boundary. I am sure it is further extended to the south and west than Tempel has it. No nebulosity seen north of *Merope*."

"1875, Oct. 29. Positive; 173.

"*Merope* looks nebulous to a distance of, say, 60″ following; then comes a dark streak, and then a narrow nebulous wisp (pos. 150°±est.) which passes through a star 14 mag., 20^s following *Merope*. This 14 mag. star is the first star following *Merope* in the same declination (approximately).

"I see nothing of Wolf's 2^d nucleus, 7^s following *Merope*."

"Observatory," vol i. p. 370. Reference to Tempel's Nebula in the *Pleiades*. It is not stated, however, what was seen with Lord Rosse's great Reflector.

Monthly Notices, vol. xl. No. 6. Mr. Common's 3-ft. Reflector. Three nebulous patches are shown, all outside of Tempel's boundary, and entirely different objects from Tempel's Nebula.

Monthly Notices, vol. xl. No. 2. Maxwell Hall. 4-inch Refractor; power 100. Drawing very similar to the one given by Tempel.

Schiaparelli, Feb. 25, 1875. "He noted it to extend from *Merope* beyond *Electra*, and as far as *Cœleno*."

Monthly Notices, vol. xl. No. 5. Dr. C. Wolf. 12-inch Refractor.

"Elle se prolonge vers *Electre* et au-delà de *Cœleno*, comme l'a dit Schiaparelli."

Dr. Engelman. Circular patch of light 11′ from *Merope* in the direction of Tempel's Nebula.

A comparison of the observations of these various authorities leaves the position, extent, and boundary of the Nebula very uncertain. It seems clear, however, that the great telescopes do not locate it in the position assigned by Tempel.

The angle of position of the axis of the Nebula with reference to *Merope*, as seen by different observers, appears to be approximately as follows :—

Tempel, Maxwell Hall, Engelman	205°
Wolf and Schiaparelli	315°
Common...	20°, 135°
Holden	...150°, 25°, 205°

In view of such great discrepancies by observers with first-class telescopes, we think it may with great propriety be asked whether these various appearances are due to real matter, or are simply an illusion.

We are strongly inclined to the opinion that the phenomena in question are due to the glow proceeding from *Merope* and neighbouring stars. With a small telescope and low power we should have to contend with the light from numerous stars in different parts of the field. As the size of the telescope is increased, the field will be diminished, and a less number of stars will be included, which might materially modify the phenomena.

In our observations of Nov. 29 and Dec. 2, the glow seemed to have a definite outline, on the north and following sides, such as might be produced by real matter; but so soon as an occulting-bar was introduced to cut off the direct light of *Merope*, the whole phenomenon disappeared : whereas, had it been due to real matter, it should have been more conspicuous when the occulting-bar was used.

Until, therefore, the observations by different observers and telescopes are better reconciled than at present—not to mention the prominent astronomers who have failed to detect any nebula—we think there is just ground to doubt the existence of any real matter in the position assigned for this Nebula.

Dearborn Observatory :
1881, *March* 29.

Ephemeris for finding the Positions of the Satellite of Neptune, 1881–82. By A. Marth, Esq.

P, angle of position of the minor axis of the satellite's apparent orbit, in the direction of superior conjunction.

a, *b*, major and minor semi-axis of the apparent orbit.

Long.=longitude of the satellite in its orbit, reckoned from the point which is in superior conjunction with the planet.

G. Noon.	P.	*a*	*b*	log. *a*	log. *b*	Long.	Diff.
1881.	°	″	″			°	°
Sept. 6	315·72	16·68	6·80	1·2222	0·8323	99·65	612·50
16	315·64	16·76	6·82	·2243	·8336	352·15	·44
26	315·53	16·83	6·83	·2261	·8342	244·59	·40
Oct. 6	315·39	16·89	6·83	1·2276	0·8342	136·99	·36
16	315·23	16·93	6·81	·2288	·8334	29·35	·33
26	315·06	16·96	6·79	·2295	·8320	281·68	·31
Nov. 5	314·89	16·97	6·76	1·2298	0·8301	173·99	·29
15	314·71	16·97	6·72	·2296	·8275	66·28	·29
25	314·54	16·94	6·68	·2290	·8246	318·57	·31
Dec. 5	314·38	16·90	6·63	1·2279	0·8213	210·88	·32
15	314·25	16·84	6·57	·2264	·8178	103·20	·34
25	314·14	16·77	6·52	·2246	·8143	355 56	·39
1882.							
Jan. 4	314·06	16·69	6·47	1·2225	0·8109	247·95	·44
14	314·01	16·60	6·42	·2202	·8077	140·39	·50
24	314·00	16·51	6·38	·2177	·8049	32·89	·54
Feb. 3	314·02	16·41	6·35	1·2152	0·8025	285·43	·60
13	314·07	16·32	6·32	·2127	·8006	178·03	·65
23	314·16	16·23	6·30	·2103	·7993	70·68	612·71
Mar. 5	314·29	16·14	6·29	1.2080	0·7986	323·39	

These values are to be interpolated for the times for which the apparent places of the satellite are required, and the position-angles p and distances s are then found by

$$s \sin (P-p) = a \sin \text{long.}$$
$$s \cos (P-p) = b \cos \text{long.}$$

The satellite moves in the direction of decreasing position-angles, and will be at its greatest elongations ("nf." in posit. $P+90°$ and distance a, "sf." in posit. $P-90°$) and at its conjunctions ("sup." in posit. P and distance b, "inf." in posit. $P-180°$) at the following hours, Gr. M. T.:—

	"sp." Elong. h		"inf." Conj. h		"nf." Elong. h		"sup." Conj. h
1881, Sept.	5	20·2	Sept. 7	7·5	Sept. 8	18·7	Sept. 10	6·0
	11	17·3	13	4·5	14	15·8	16	3·1
	17	14·3	19	1·6	20	12·9	22	0·2
	23	11·4	24	22·7	26	10·0	27	21·2
	29	8·5	30	19·8	Oct. 2	7·0	Oct. 3	18·3
Oct.	5	5·6	Oct. 6	16·9	8	4·1	9	15·4
	11	2·7	12	13·9	14	1·2	15	12·5
	16	23·8	18	11·0	19	22·3	21	9·6
	22	20·9	24	8·1	25	19·4	27	6·7
	28	18·0	30	5·3	31	16·5	Nov. 2	3·8
Nov.	3	15·1	Nov. 5	2·4	Nov. 6	13·6	8	0·9
	9	12·2	10	23·5	12	10·7	13	22·0
	15	9·3	16	20·6	18	7·9	19	19·1
	21	6·4	22	17·7	24	5·0	25	16·2
	27	3·5	28	14·8	30	2·1	Dec. 1	13·3
Dec.	3	0·6	Dec. 4	11·9	Dec. 5	23·2	7	10·4
	8	21·7	10	9·0	11	20·3	13	7·5
	14	18·8	16	6·1	17	17·4	19	4·6
	20	15·9	22	3·2	23	14·5	25	1·7
	26	13·0	28	0·3	29	11·6	30	22·8
1882, Jan.	1	10·1	Jan. 2	21·4	Jan. 4	8·6	Jan. 5	19·9
	7	7·2	8	18·5	10	5·7	11	17·0
	13	4·3	14	15·5	16	2·8	17	14·1
	19	1·3	20	12·6	21	23·8	23	11·1
	24	22·4	26	9·6	27	20·9	29	8·2
	30	19·4	Feb. 1	6·7	Feb. 2	18·0	Feb. 4	5·2
Feb.	5	16·5	7	3·7	8	15·0	10	2·3
	11	13·5	13	0·8	14	12·0	15	23·3
	17	10·5	18	21·8	20	9·1	21	20·3
	23	7·6	24	18·8	26	6·1	27	17·3
March	1	4·6	March 2	15·8	March 4	3·1	March 5	14·3

In reference to his Paper, "Telegraphic Determination of the Longitude of Shanghai," *Monthly Notices*, December No. *ante*, pp. 64–67, Dr. L. S. Little writes:—

"The object and public use of the determination of the longitude of the Observatory was, that by it I determined the longitude of the British Consular Flagstaff, which is the point of departure of the charts, &c. This point is $1^{s}\cdot02$ E. of Observatory, and the only point in China determined telegraphically"

Observations of the Companion of Sirius, made at the U.S. Naval Observatory, Washington.

(*Communicated by Rear-Admiral John Rodgers, U.S.N., Superintendent.*)

Date.	Sid. Time.	p.	s.	Weight.	Observer.
	h	°	″		
1881, Jan. 28	7·5	43·1		3	Hn.
Feb. 17	6·9	46·0	10·63	3	F.
19	6·5	46·4	9·96	3	F.
25	8·8	43·5	10·83	4	Hn.
March 7	7·0	48·4	10·02	3	F.
10	7·0	46·2	10·01	2	F.
11	6·5	46·0	10·02	2	F.
14	7·7	48·3	10·01	2	F.
26	6·9	45·3	9·99	2	H.
27	7·0	44·8	10·17	2	H.
28	6·9	44·2	9·88	3	H.
April 7	8·0	44·5	10·01	2	H.
10	8·2	46·3	10·00	2	H.
11	8·5	46·7	9·96	3	H.

Results.

	p.	s.	Observers.
	°	″	
1881·117	43·30	10·83	E. S. Holden.
1881·174	46·88	10·108	E. Frisby.
1881·256	45·30	10·002	A. Hall.

Errata.

P. 334, last line but one, last column, *for* + 1·59 *read* + 1·95.

„ 357, ζ *Aræ*, in column for Number of Observations at Lower Transit, *insert* 6, and three lines lower down, *for* Star' *read* Star's.